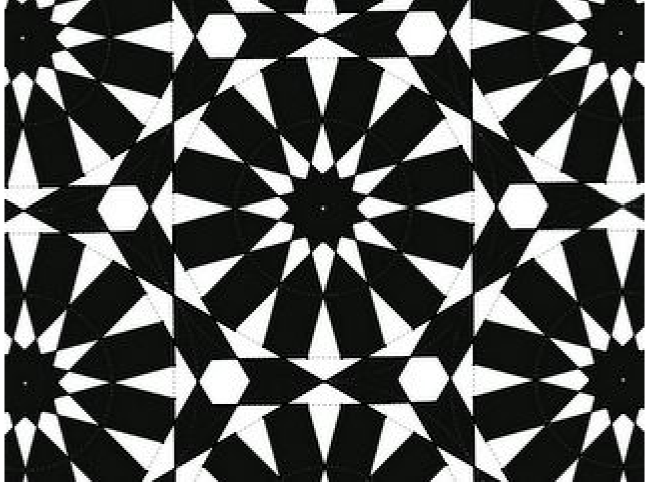
Thomas Christie

Reconstructing Rosette-based Islamic Geometric Patterns



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2006

1.1 *Introduction*

For centuries a major component of Islamic art has been the construction of space-filling geometric patterns. There are no known formal rules prescribing the construction or final design of these patterns, and as a result this art form exhibits enormous variation of design. Nevertheless, one type of pattern is recognizably distinct: patterns containing rosettes, or *star patterns* (see page A1 of the appendix, attached). The rosette is a motif found in Islamic geometric patterns worldwide, and has been called “the most typically 'Islamic' of all star motifs.”[1] Though rosettes in various star patterns are visibly similar, they also exhibit a great deal of variation, as does their position relative to each other in the plane. The ultimate goal of this paper is to develop rules and parameters for constructing rosettes that are flexible enough to incorporate variations found in historical examples, and formulate a technique for constructing completed star patterns that correspond to patterns found in the literature.

Throughout history, star patterns have been designed by master craftsmen who have kept the secrets of their designs closely guarded.[2] As a result, and despite the worldwide prevalence of these patterns, there is an extraordinarily limited amount of literature concerning the details of their construction. Several design algorithms have been proposed by various scholars, primarily computer scientists, describing both the construction of rosettes and how to piece them together to tile the plane.[3] Unfortunately, all of the algorithms I have found thus far require extensive computation and would be extremely difficult to implement without aid of a computer. The original designers these patterns clearly did not utilize computers, so they must have used a simpler method of construction. In this paper I introduce an algorithm I have developed that can be implemented with a ruler, compass and protractor, that can be used to construct an enormous range of historical star patterns. Due to its ease of use and ability to produce patterns that exactly match those in historical examples, I believe there is a high probability that a very similar algorithm was used by these patterns' original designers. In this sense, I consider my project a success.

As it is described in this paper, the algorithm I propose can be used to create many star patterns in which every rosette is the same size and has the same number of petals. Many Islamic patterns have rosettes of different sizes and petal numbers in the same pattern, so there is still a great deal of room for future research in this area.

In the first part of this paper I introduce the rosette motif and elaborate on its properties. Next I describe a rosette construction method found in a book by Abas and Salman, which creates a limited range of rosettes. I then introduce a generalized and more versatile algorithm, which I call the *circle method*, and describe the details of its implementation and how to use it to create rosettes that match a large array of historical examples. Lastly, I discuss the mathematics of tiling the plane with circles, and describe how to use this knowledge in conjunction with the circle method to tile the plane with rosettes to form complete star patterns.

2.1 *Rosettes*

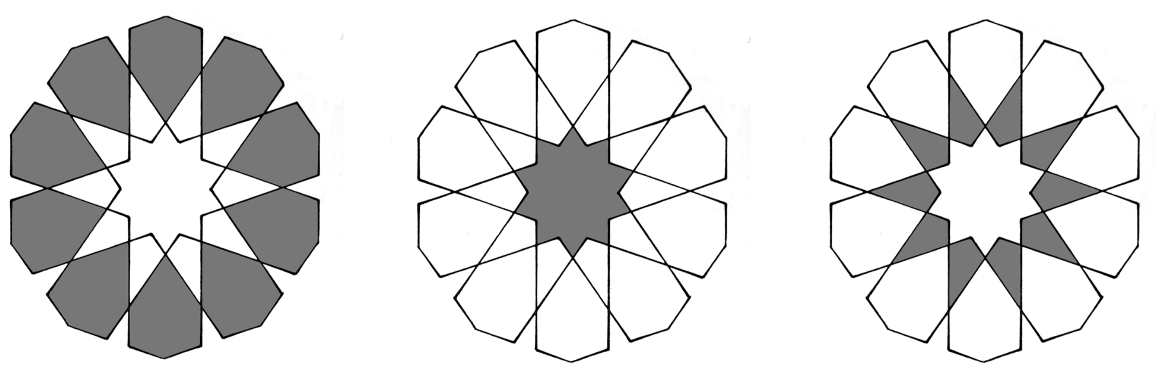

Figure 1: Highlighted, from left to right: petals, center, diamonds.

Generally speaking, a rosette is an *n*-sided star shape surrounded by *n* quadrilaterals and *n* hexagons, as shown in Figure 1. While recognizing rosettes is easy, defining them is more difficult. Nevertheless, all rosettes have certain common attributes. Rosettes have three parts, highlighted in Figure 1, which I will call the petals, the center, and the diamonds, respectively. Moreover, in rosettes and star patterns in general, only straight lines are used in construction. When two straight lines meet, they are either both truncated and thus create a "bend," such as on the outer- and inner-most edges of the rosettes in Figure 1, or cross and continue without bending at their point of intersection. This means that any "node" or line intersection has a degree of either two or four. I assume that the original designers created patterns in this way to make them color-able with two colors, but that is a hypothesis to be explored in another paper.

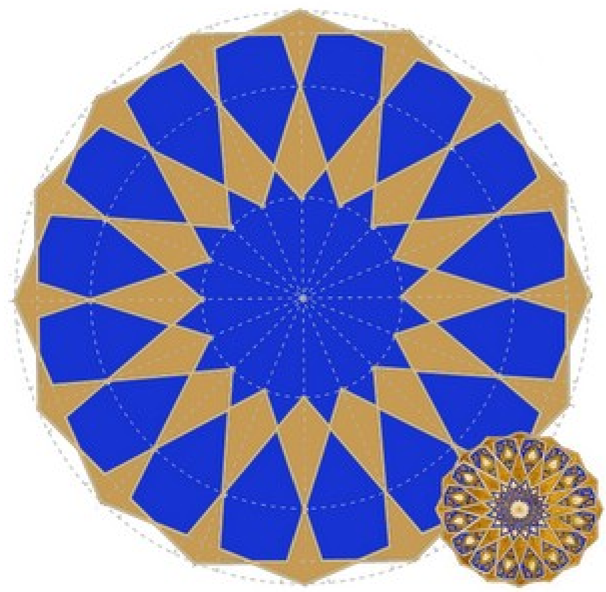

Figure 2: A 16-sided rosette with non-parallel petal sides.

Rosettes can vary in a number of respects. Most importantly, they are typically characterized by their number of petals. This number ranges from six to 96, though rosettes with more petals are possible to construct in theory. The rosette pictures above has ten petals.

Another variation commonly seen concerns the shape of the petals. Notice that the two lines that construct the sides of the petals in Figure 1 are parallel. For example, in the top-most petal in each rosette in Figure 1 on the previous page, these lines are the lines that point straight up and down and form the left and right edge of the petal. These

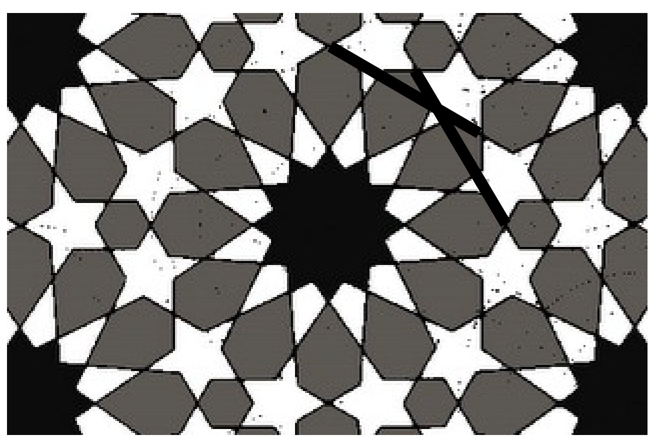

Figure 3: A 12-sided rosette that is not "flat." This images is adapted from Bourgoin (see "Sources Used..."), page 68.

lines continue inwards until reaching the center, and also compose the edges of the diamonds. I call these lines "foundation lines," because they form the foundation of all but the outermost edges of the rosettes. These lines are groups in pairs of two, and each pair defines the sides of a petal. Not all rosettes feature parallel foundation lines, however. Figure 2 features a rosette with non-parallel lines, and notice that the center of the rosette is large relative to the centers in Figure 1.

The final variation commonly found in rosettes concerns the outermost edges of the rosette petals. In Figure 1, the edge line from one petal "points" at the edge line of the adjacent petal. That is, if one extended an edge line it would overlap entirely with the edge line of the next petal. I term a rosette with such properties "flat," because the outer points of the rosette are relatively flat, and in fact become flatter as the numbers of petals a rosette has increases. The rosette in Figure 3 is part of a larger pattern, but notice that the outermost edge of the petals does not point at the edge of the next petal over, but at the edge of the petal *two away* from itself. I have drawn a line on the rosette to illustrate what am referring to. The rosette in Figure 2 is also flat, if one disregards the outermost embellishments.

Any algorithm for constructing rosettes must therefore be flexible enough to incorporate the three variations mentioned thus far: a range of numbers of petals, non-flat edges, and both parallel and non-parallel foundation lines. I describe such an algorithm below.

2.2 *The Circle Method – A Hint*

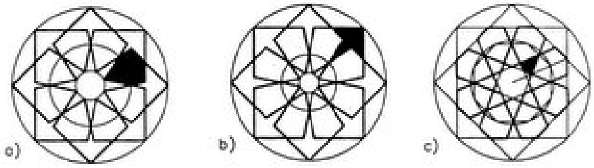

Figure 4: Illustrations by Abas and Salman (see footnote 5)

As mentioned above, a great deal of the suggested algorithms for constructing rosette-based geometric patterns require a computer for their implementation. While some work quite well[4], it is highly unlikely that such methods were used originally. While trying to discover easier ways to construct rosettes and star patterns, I discovered the image in Figure 4 in a book by Syed Jan Abas and Amer Shaker Salman called *Symmetries of Islamic Geometrical Patterns*[5] with the accompanying text:

> *The shapes in [Figures 4a and 4b] have been obtained quite similarly by introducing two new concentric circles in Fig 1.4b [which is two squares embedded in a circle]. These are again divided symmetrically to yield eight points... Joining the points as shown, produces some secondary polygons which have been filled in black. They represent yet other shapes which occur regularly and characterize Islamic patterns.*
>
> *The radii of the two inner circles are arbitrary and can be varied to alter the size of the inner 8-pointed star shape and the associated polygons. Figures [4a and 4b] show two variations...*
>
> *Figure [4c] represents a special case of the shapes that are produced when the radii of the circles are varied. Here, only one of the two circles is allowed an arbitrary radius. The radius of the innermost one is forced to have a fixed size in relation to the one in the middle... The shape is completed by joining lines through the mid-points of the octagon and and through making use of the sides of the two squares defined by the outer circle, as shown...*
>
> *It is easy to generalize the above procedure by choosing to inscribe more than two squares in the outer circle.*

It is the goal of this paper to expand on this information and generalize it as much as possible to develop a flexible method for creating rosettes. I will describe the application of this method in the step-by-step construction of an 8-sided rosette as shown in Figure 5. Then I will demonstrate the ways I have expanded and generalized this process to enable the construction of a variety of rosettes.

2.3 *The Circle Method*

The most commonly found rosette has 8 sides, and its construction begins by embedding two squares into a circle.[6] This is equivalent to drawing a circle and placing 8 equidistant points on its circumference, then drawing line segments connecting every second point. The construction process is as follows, and each number corresponds to an image in Figure 5:

1. Draw a circle with p equidistant points on its circumference. In the illustration in Figure 5, $p = 8$.

2. Draw a line segment connecting every point with the q^{th} point away. In this example, $q = 2$. This creates a design with the Schläfli symbol {8/2}, which will be discussed below.[7]

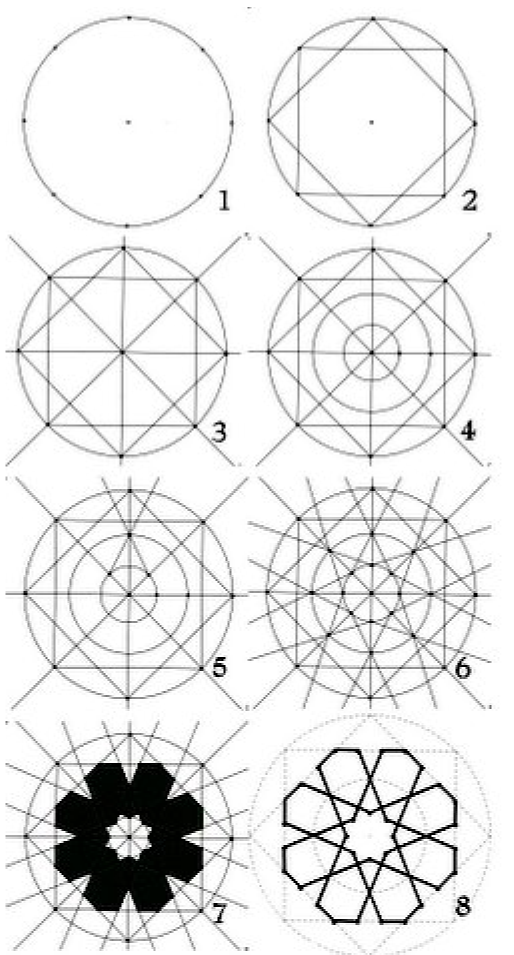

Figure 5: The construction of an eight-sided rosette.

3. Determine what intersections will form the edge of the petals. In this case, each intersection of the two squares will form the outer point of a petal. Variations on this choice are discussed below. Draw rays from the origin that pass halfway between the petal tips. I call these *construction lines*.
4. Draw two circles with different radii in such a way that they lie *inside* the petal tips. I call these the *inner* and *middle* circles.
5. & 6. At each intersection of the inner circle and the construction lines, draw the origin of two rays. These rays should pass through the intersection of adjacent construction lines with the middle circle. I call such rays *foundation lines*.
7. & 8. Darken the lines that compose the finished rosette. Alternatively, erase all other lines.

Notice that the finished rosette is nested within in an outer circle. In the next section, I will demonstrate how to tile the plane with circles. Rosettes can then be placed in these circles as shown to create a complete pattern. Both parts of this construction can be performed using only a compass, protractor and ruler, and it therefore stands a good chance of being the same algorithm used by the original creators of these patterns.

2.4 *Variations on this Theme*

By generalizing certain steps of the above construction, a variety of rosettes can be constructed. This generalization accounts for the variations described in Section 2.1. The most important variation concerns the number of petals in a rosette. This number is ultimately determined by the number of equidistant points, p, initially placed on the outer circle in Step 1 of Figure 5. Following the remaining steps will lead to a rosette with p sides. The second variation concerns the lines drawn in Step 2. Lines can be drawn that connect every point with the points adjacent to it (as in the top image in Figure 6), every two points, three points, or in general every q points. Each choice creates an "inner polygon" within our original circle, within which the rosette is constructed. For an even p, connecting each point with the point $p/2$ away will merely create a point instead of a polygon, and connecting every p/2+1 points is equivalent to connecting every p/2-1 points, so effectively we must choose q such that $1 \geq q > p/2$. For an odd p, we then have $1 \geq q > (p\text{-}1)/2$.

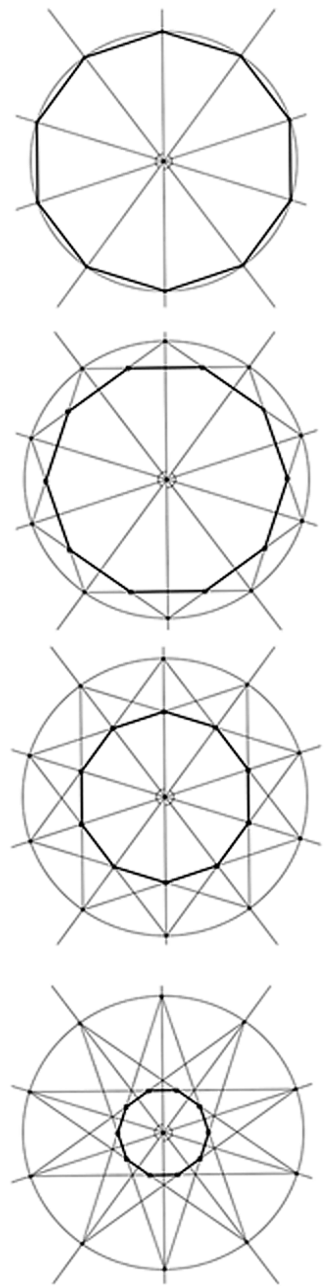

Figure 6: Internal decagons created by joining every point to the 1, 2, 3 and 4 away from it.

Each combination of p and q will yield a distinct rosette in our final pattern. The idea of connecting every q points in a polygon with p points was studied by Ludwig Schläfli, and the notation {p/q} is called a "Schläfli Symbol."[7] I will use this notation for the remainder of this paper. In Figure 6 I demonstrate {10/1}, {10/2}, {10/3}, and

{10/4}, each of which creates a inner polygon of a different size relative to the outer circle.

So far I have only constructed rosettes inside this inner polygon. Doing so creates the "flat" rosettes described above. Non-flat rosettes are often found in the literature, and the method of construction introduced in this section must account for such variation. In nearly all non-flat rosettes, the outer edges of the petals do not "point" to the adjacent petal, but to a petal further away. In Figure 3, each petal points at the petal two away. The same phenomenon is produced by constructing the rosette so that the outer edges of the petals are not defined by the inner polygon, but by the angles created when $q > 1$. For example, the lines in the second drawing in Figure 6 can be viewed as a decagon in a circle, or as two pentagons. It is possible make the edges of the pentagons form the rosette petals, and draw construction lines in between them, as in the third drawing. In the third and fourth drawings, the rosette could be extended even further, creating a more acute angle on the tips. As mentioned, the lines that define such extensions only exist with $q > 1$, since the {10/1} in Figure 6 provides no lines with which to construct such extensions within the outer circle.

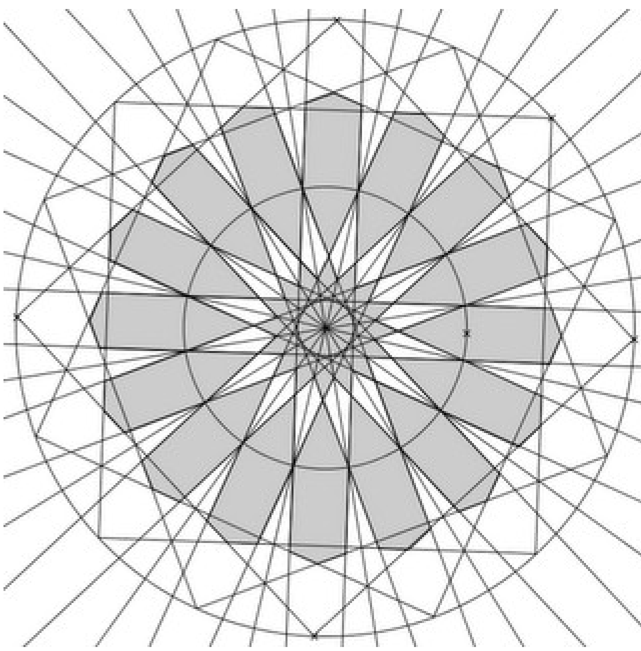

Figure 7: A rosette of the form {16/4} with an extension of degree 1.

I term a rosette created within the innermost polygon, called "flat" elsewhere in this paper, a design with a degree 0 extension. A rosette drawn using the next available angles, moving outward, has an extension of degree 1. Figure 7 demonstrates a {16/4} rosette with such an extension. In page A2 of the appendix, I have drawn rosettes based on {16/4} with extensions of degree 0 through 3. In general, a {*p*/*q*} rosette can have extensions varying from 0 to *q-1*.

The final common variation is the shape of the sides of the petals, and whether or not they are parallel. The method of construction so far requires the petal lines to be drawn as rays through certain points on the inner and middle circle. Varying the size of the inner circle forms petals of different shapes. A large inner circle creates petals that are thicker towards the center of the rosette (as in Figure 2), while a small inner circle makes petals thicker towards the center. The drawings in Figure 8 demonstrate this clearly.

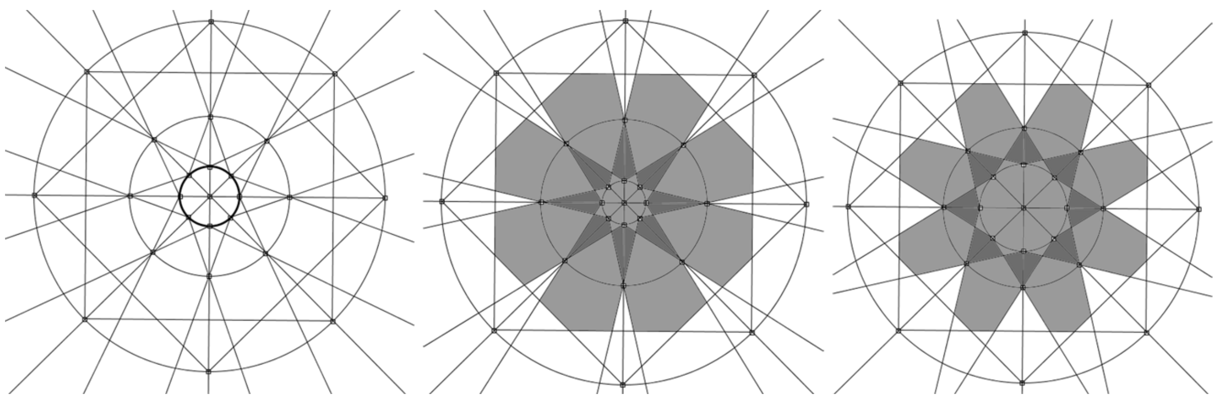

Figure 8: Varying the size of the inner circle creates petals of different shapes.

As the petals are defined by lines intersecting the inner and middle construction circles, the ratio of these circles defines the shape of the petals. This ratio can be calculated using basic geometry, but the result is a sine function that gives little aid when constructing rosettes by hand. In fact, when constructing even-number rosettes with petals with parallel sides the inner circle becomes unnecessary and can be omitted. Simply drawing lines that connect the intersection of a construction line and the middle circle with the same intersections *adjacent to* the opposite intersection will yield the same results. Figure 4.C demonstrates this, as does the {16/4} rosette in Figure 7. The lines that compose the sides of one petal continue through the center and form the sides of the petal directly opposite. Odd-numbered rosettes do not have this symmetry, and petal sides are best constructed using rays as originally described.

Lastly, the size of the middle circle defines the width of the petals. This is a byproduct of the fact that the petal sides are defined by the intersection of construction lines and the middle circle. This relationship is intuitive when one notices that the petals all touch adjacent petals along the middle circle, as shown in Figure 7. Thus, a larger middle circle results in thicker petals.

The above algorithm and variations described can produce an endless variety of rosettes. Variables accounted for include numbers of petals, the size of the rosette in relation to the outer circle, the degree of the rosette's extension, the thickness of the petals and whether or not the rosette has petals with parallel sides. With this in mind I will turn to the mathematics of tiling the plane with circles, and finish with a description of how to use this information to create a finished pattern.

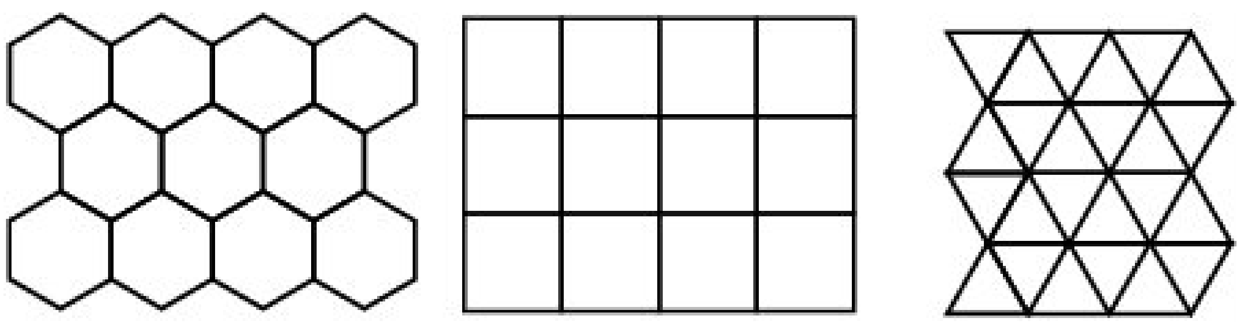

Figure 9: Drawings of the three regular tilings of the plane: hexagonal, square and triangular. Cited in Footnote 8.

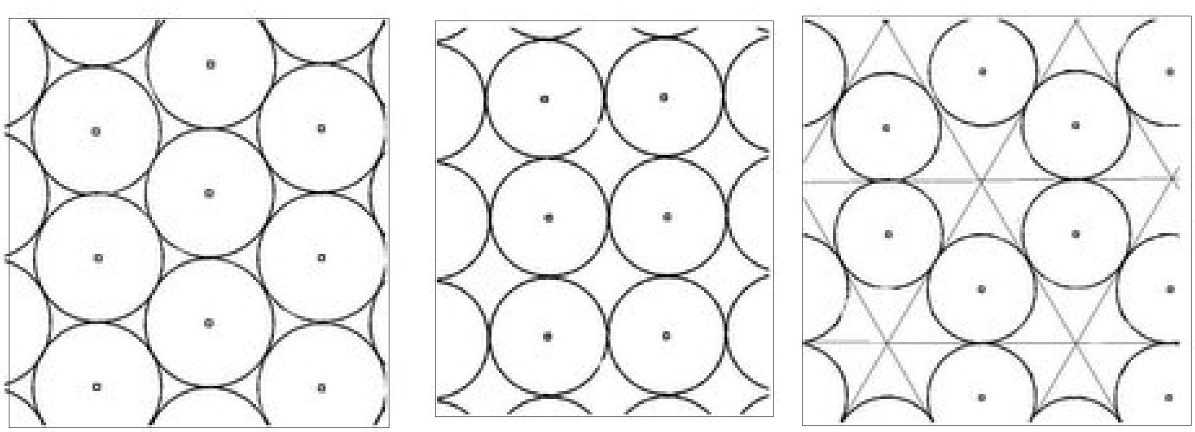

Figure 10: Corresponding tilings of the plane with circles.

3.1 *Tiling the Plane*

It is easily proved that only three regular polygons will tile the plane: triangles, hexagons and squares.[8] These tilings are shown in Figure 9. To tile the plane with circles, we draw circles in each polygon in such a way that they touch the sides of the polygon. We then erase the original polygons as done in Figure 10. Due to the symmetry of these tilings, each circle touches the circles in each of the adjacent polygons at a single point. We are therefore left with three distinct "tilings" of the plane with adjacent circles.

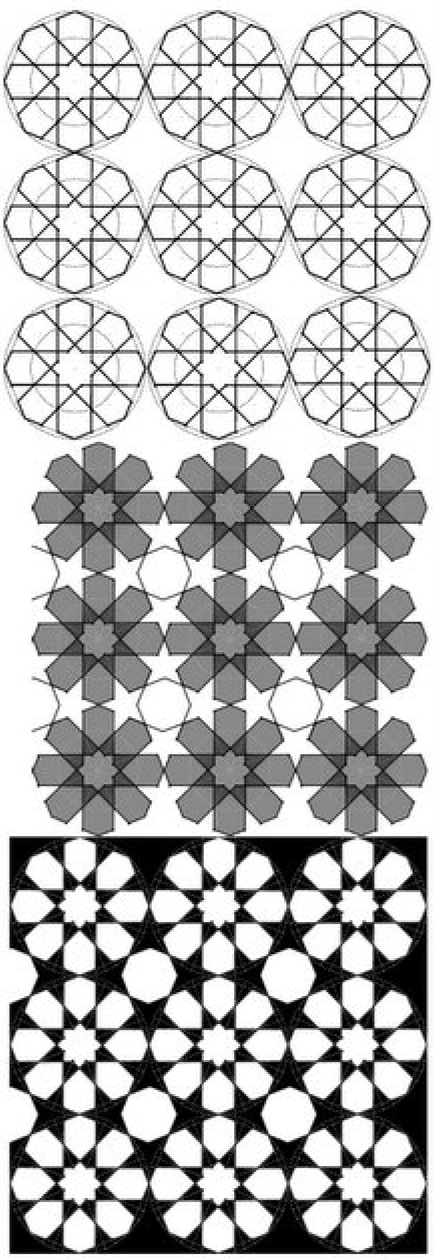

Figure 11: Creating a finished pattern: Place circles, draw filler lines, erase construction lines and then color.

3.2 *Filling Circles with Rosettes*

Using the Circle Method, described in Section 2.3, we now place rosettes inside each of the circles on the plane. Examples of star patterns that seem to be constructed using these three tilings are given in the appendix, page A1. In patterns where all rosettes have the same number of petals, and such patterns are the primary focus of this paper, each rosette has both translational and reflective symmetry with its adjacent rosettes. This means that if rosettes were placed in the circles above, it would be possible to cut them into "pie piece" sections, with one section for each adjacent circle. Effectively, this means that rosettes placed in the circles must have a number of petals that is a multiple of six, four or three respectively.

Once rosettes are placed in the circles, "filler lines" must be drawn to fill the space between the circles and connect the

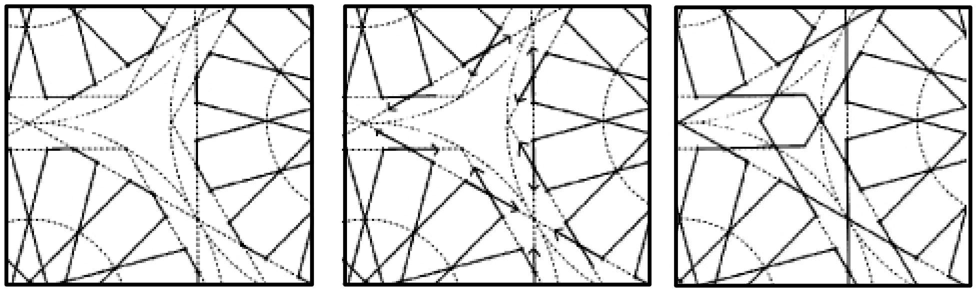

Figure 12: The process of creating filler lines to connect the rosettes.

lines of the rosettes. The construction lines and circles are then erased. This is demonstrated in Figure 11, using a square-tiling-based, {8/1} pattern.

There does not appear to be a set rule for creating "filler lines," so long as they follow the two basic rules of all star patterns: the lines drawn must either truncate or continue straight without bending at each node. Filler lines are almost always continuations of the two lines that make up the outermost edge of the petals.

The series of required steps is demonstrated in Figure 12, above. In Figures 11 and 12, the construction of filler lines is simple and deterministic. When the rosettes are far apart, such as when q is quite large in a {p/q} pattern, a great deal of creativity may be involved in constructing filler lines. In the examples I have seen, there seems to be no continuity from one pattern to the next regarding the way these filler lines are constructed. Using the triangle tiling (see Figure 10, the rightmost image) leaves a good deal of space between each hexagonal group of circles which can then be filled in any way, providing the required symmetry is maintained. The top right image on page A1 of the appendix is a good example of such a pattern. A series of images comparing the tilings produced by {12/1} and {12/2} rosettes can be seen on pages A4 and A5.

4.1 *Conclusion*

The algorithm I propose in this paper can be used to reconstruct (or construct anew) star patterns that have all of the following properties:

1. All rosettes in the pattern have the same number of petals.
2. All rosettes in the pattern are the same size.
3. The rosettes are in hexagonal, triangular or square formations.
4. The number of petals on these rosettes are multiples of 6, 3 or 4 respectively.

My algorithm allows for "extensions," or petals with varying edge-angles, and petals with non-parallel sides. It also allows for completed patterns with a varying amount of space between rosettes, which is controlled by the $\{p/q\}$ parameter described above. A great deal of star patterns have properties 1-4 and the variations just described, and I have used the algorithm to reconstruct many of these, which was the original goal of my project. I demonstrate the construction of complete patterns in Figures 5 and 11, and in pages A4 and A5. I therefore consider my inquiry successful.

However, I have come across many beautiful patterns that cannot be created with the circle method as described in this paper. There is therefore a good deal more research to be done in this area.

4.2 *Future Research*

The circle tilings used in this paper are based on regular triangular, square and hexagonal tilings of the plane. Circle tilings created from rectangular tilings could be explored, as could non-square grid tilings. Using a non-square grid tiling with the correct angles may be conducive to irregular numbered rosettes, such as 10 or 14. In his research on this subject (see footnote 3), Craig Kaplan describes regular and irregular tilings of the plane with polygons. It may be possible to adapt the circle method to be useful with such tilings.

I am most interested in patterns that incorporate rosettes of several different numbers. I have done preliminary research in this area, and have strong suspicions that the circle method can be applied usefully, especially regarding patterns containing very different rosettes (such as

16- and 8-petal rosettes). So far I have no systematic way of knowing how to create such a pattern from scratch. There is still much to be done, and this is an exciting topic for future study.

Sources of Patterns Used in Research:

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Bourgoin, J. Arabic Geometrical Pattern and Design. Dover Publications, 1973.

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Critchlow, Keith. Islamic Patterns. Rochester: Inner Traditions International, 1976.

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Wade, David. Pattern in Islamic Art. Woodstock, NY: The Overlook P, 1976.

Works Cited

1 Lee, A J. "Islamic Star Patterns." Muqarnas (1987).

2 Kaplan, Craig S. Computer Generated Islamic Patterns. University of Washington. <http://www.cgl.uwaterloo.ca/~csk/washington/tile/papers/kaplan_bridges2000.pdf>.

3 See Kaplan, Computer Generated Islamic Patterns,
and
Lee, "Islamic Star Patterns"
and
Kaplan, Craig S. Computer Generated Islamic Patterns. University of Washington. <http://www.cgl.uwaterloo.ca/~csk/washington/tile/papers/kaplan_bridges2000.pdf>,
and
Kaplan, Craig S. Computer Graphics and Geometric Ornamental Design. Diss. Univ. of Washington, 2002. <http://www.cgl.uwaterloo.ca/~csk/phd/kaplan_diss_full_screen.pdf>. And

4 Particularly the work done by Craig Kaplan. He has even created an applet called "Taprats," which creates star patterns based on various polygonal tilings of the plane, which is available on his website: http://www.cgl.uwaterloo.ca/~csk/washington/taprats/index.html

5 Abas, Seyed J., and Amer S. Salman. Symmetries of Islamic Geometric Patterns. Singapore: World Scientific Co., 1995. 16-17.

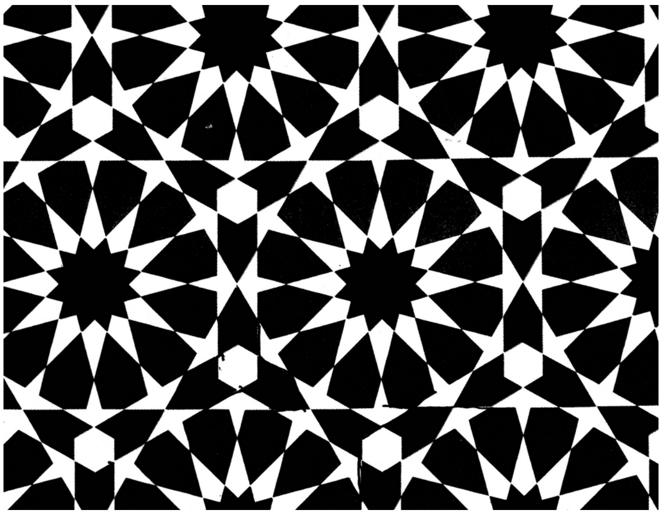
6 For a discussion of the importance of this motif in Islamic geometric design, see Abas and Salman, 14-16.

7 "Schläfli symbol." Wikipedia, The Free Encyclopedia. 4 Apr 2006, 20:27 UTC. 11 Apr 2006, 04:06 <http://en.wikipedia.org/w/index.php?title=Schl%C3%A4fli_symbol&oldid=46977224>.

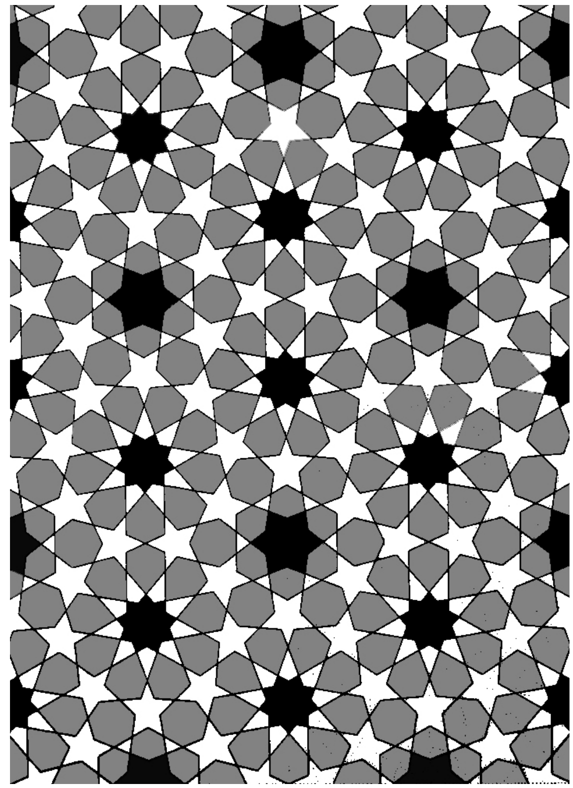
8 Eric W. Weisstein. "Regular Tessellation." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/RegularTessellation.html
Note that there are several other ways to tile the plane with both regular and irregular polygons, and creating star patterns from these tilings using the circle method would be a good area for future research.

Appendix

Example Star Patterns



[1]



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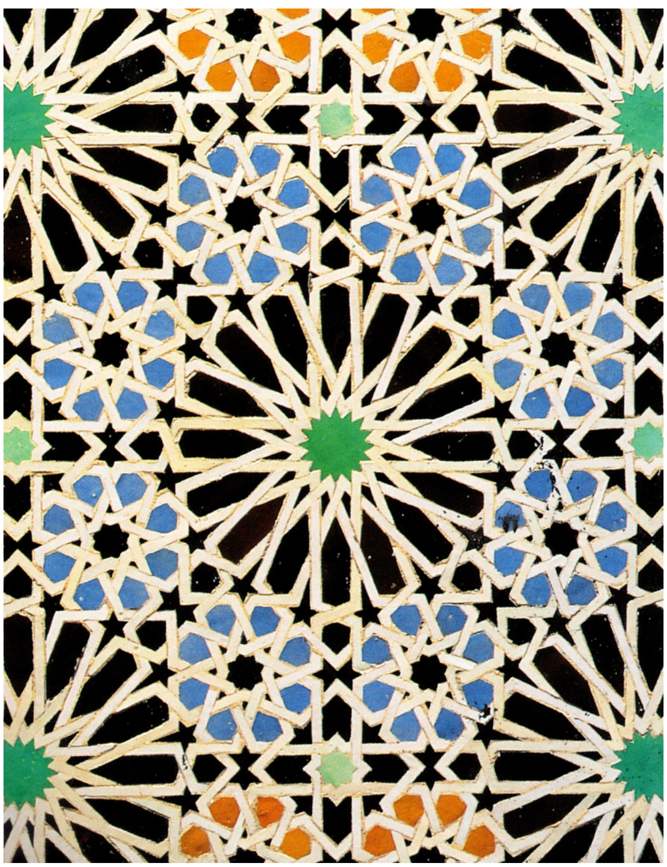

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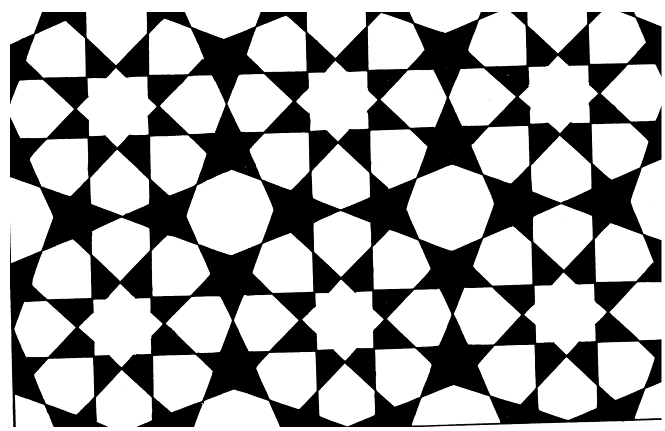
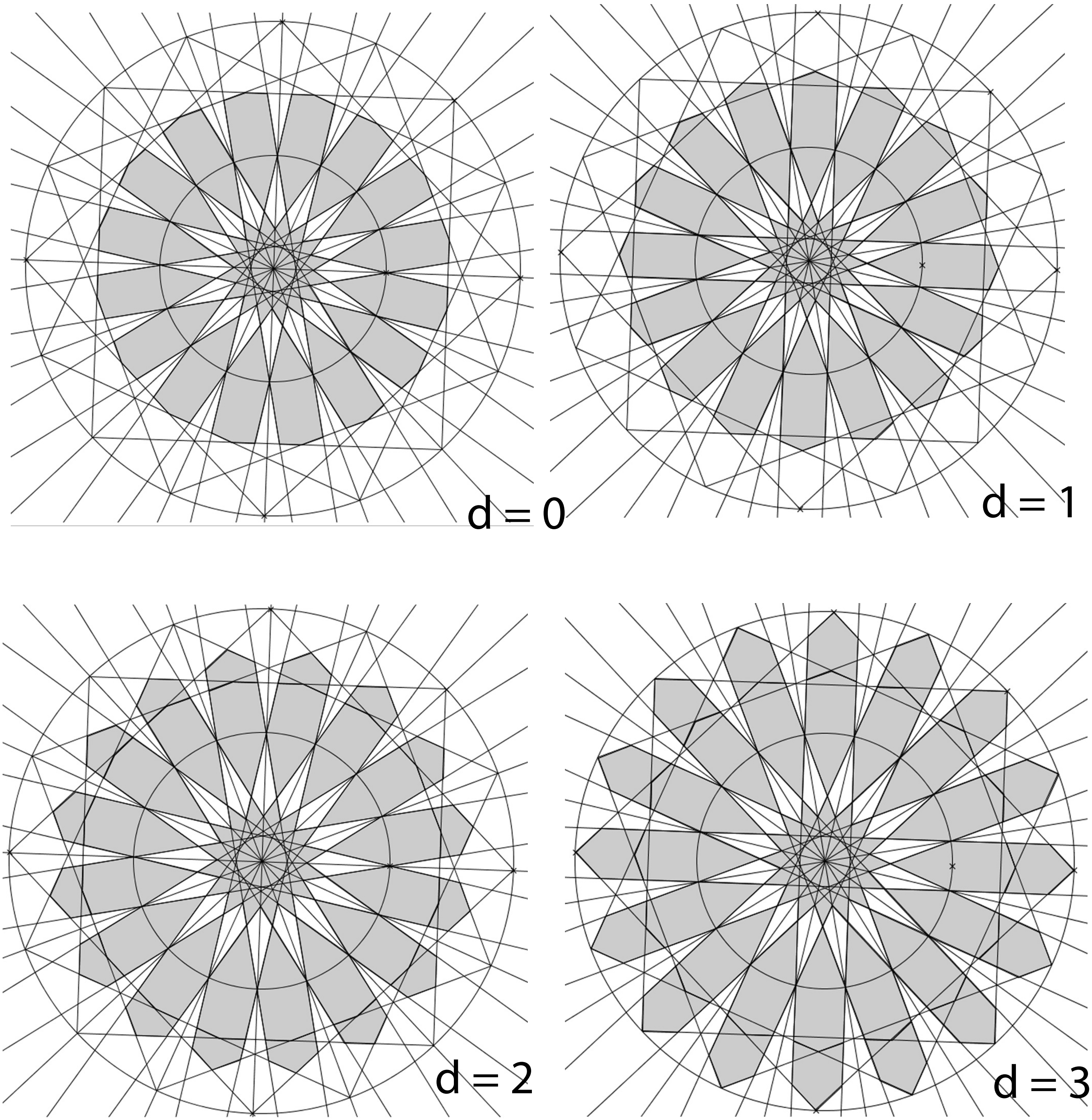


[4]

[1] Edited from Wade, David. Pattern in Islamic Art. Woodstock, NY: The Overlook P, 1976. 59.
[2] Edited from Bourgoin, J. Arabic Geometrical Pattern and Design. Dover Publications, 1973. 76.
[3] Castera, Jean-Marc. Arabesques: Decorative Art in Morocco. ACR Edition, 1999. 166.
[4] El-Said, Isaam, and Ayse Parman. Geometric Concepts in Islamic Art.
London: World of Islam Festival Company Ltd., 1976. 44.

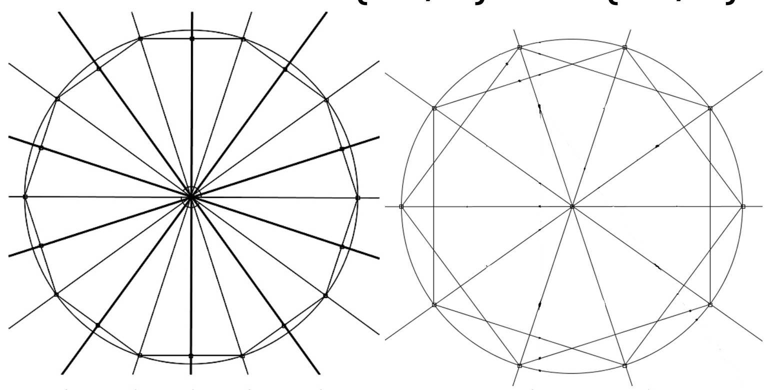


{16/4} rosettes with extensions of degrees 0, 1, 2 and 3

Construction of {10/1} and {10/2} rosettes

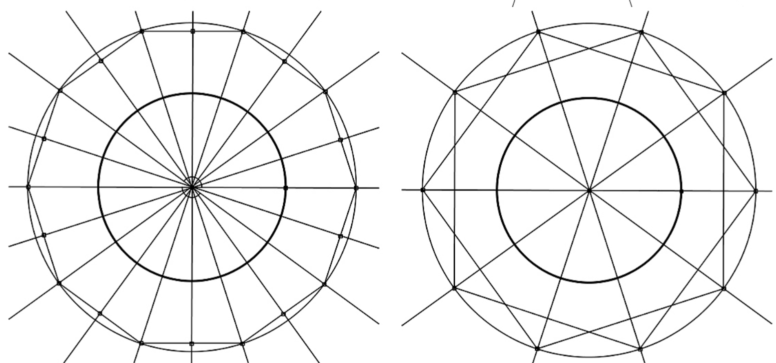
Christie A3

Step A



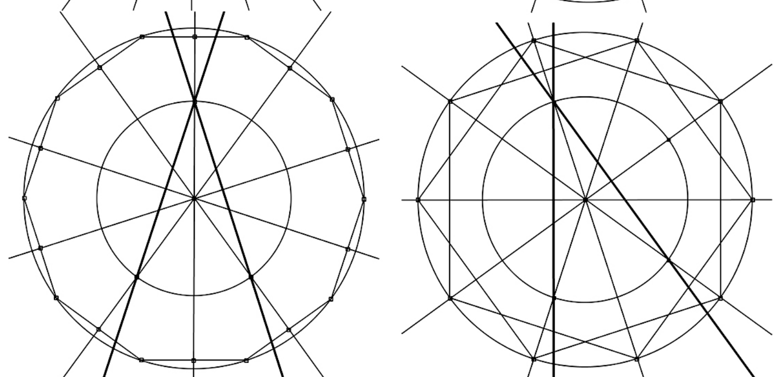
Step A: {10/1} and {10/2} are drawn in the outer circle. Extra lines are added on {10/1} that will pass half-way through the petal tips.

Step B



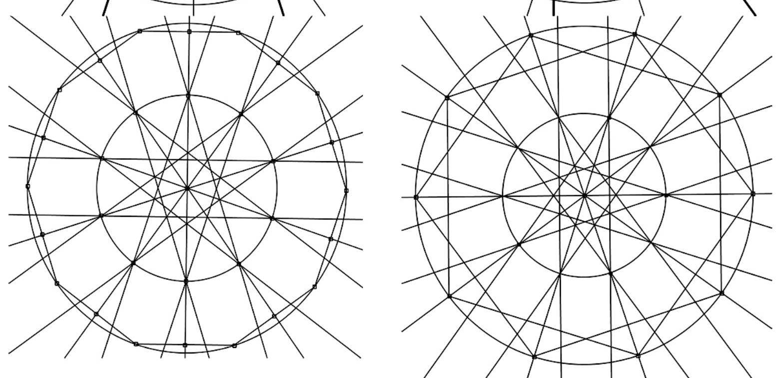
Step B: The inner circle is added. The size is arbitrary, as long as it lies within the p-gon. The inner circle is left out so the petals will have parallel sides.

Step C



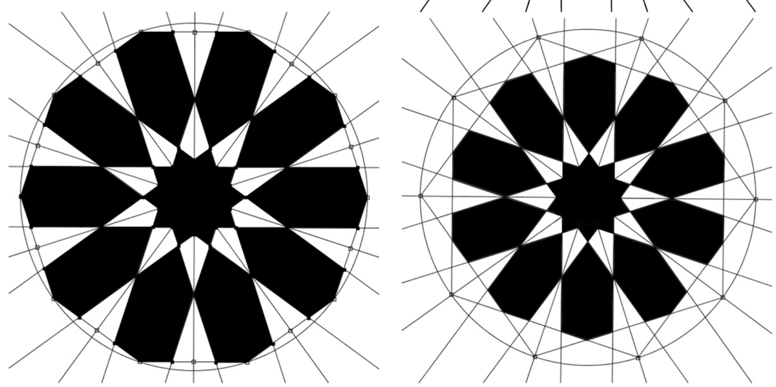
Step C: Unused lines are erased on the left. The first petal lines are drawn.

Step D



Step D: All of the foundation lines are drawn in.

Step E

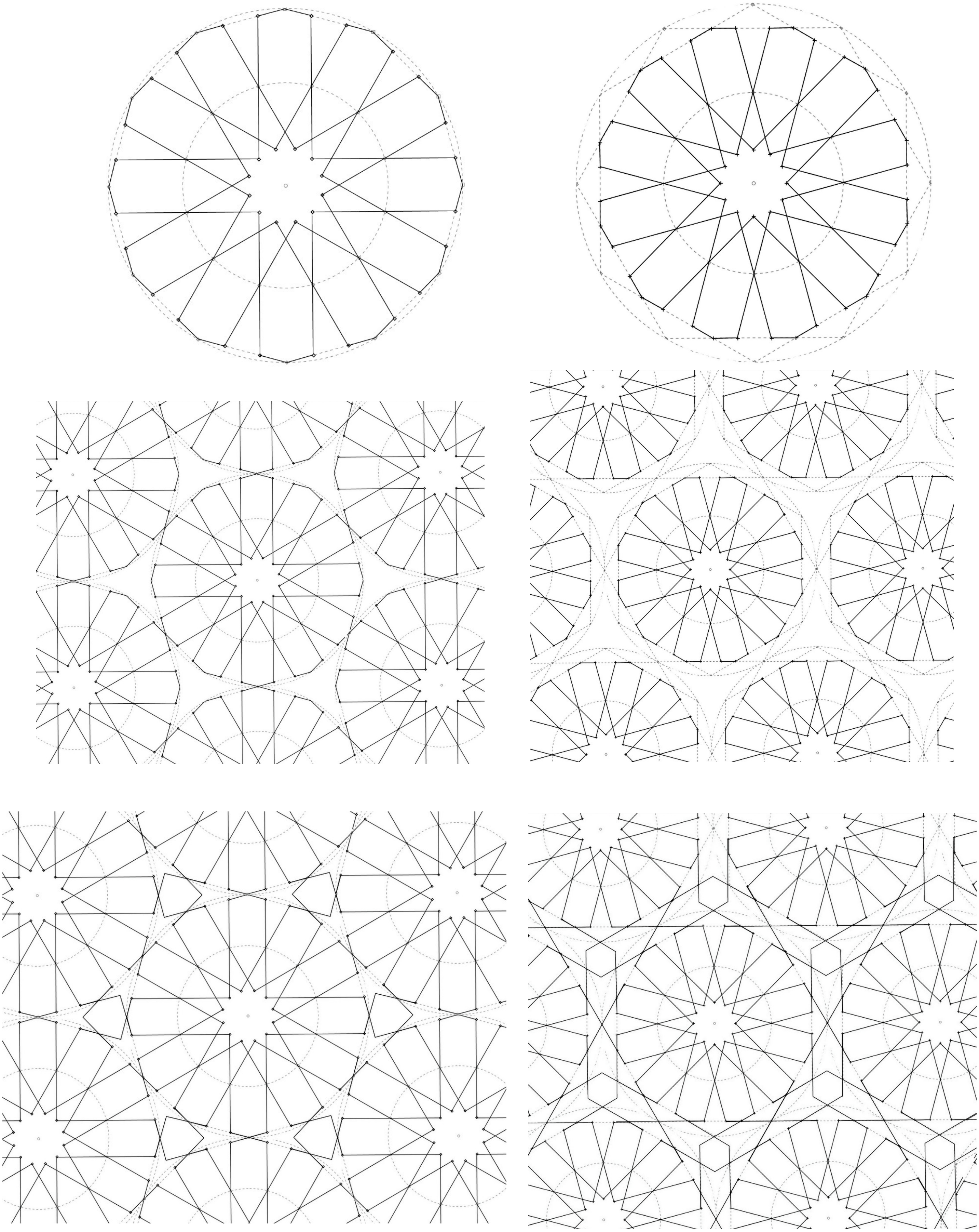


Step E: The rosettes are colored in. They are the same rosette, but are different sizes relative to the size of the outer circle

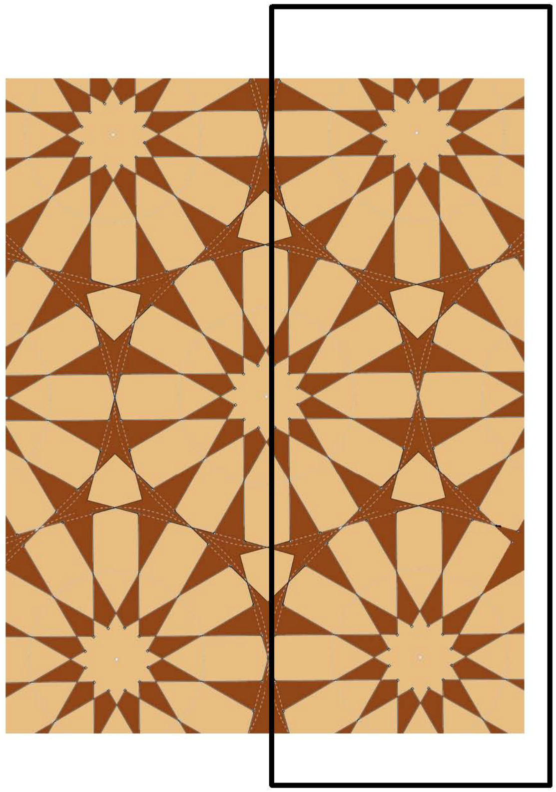
{10/1} {10/2}

Construction of {12/1} and {12/2} patterns.

Christie A4

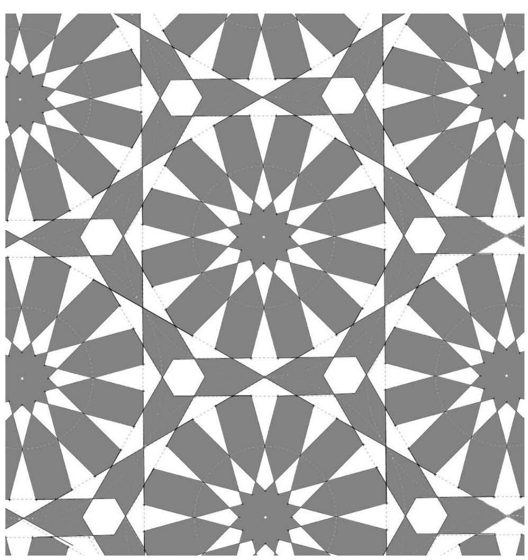


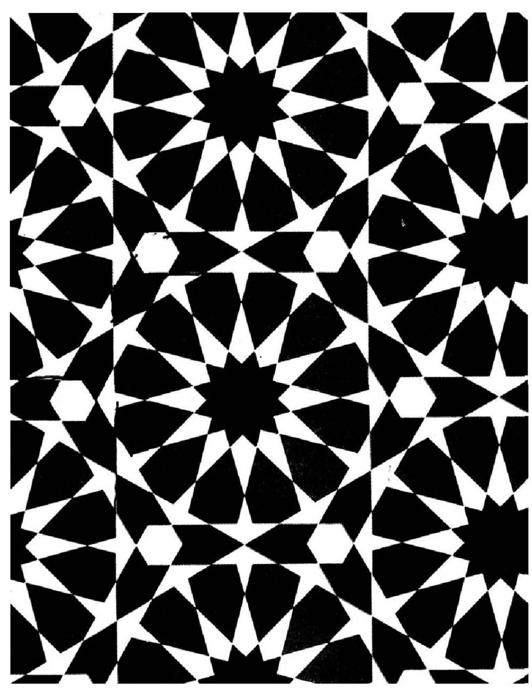
{12/1}



[1]

{12/2}





[2]

Rotated 30˚, these patterns match patterns found in the literature.

[1] Doors, Pair. The Metropolitan Museum of Art, New York. Works of Art: Islamic Art. <http://www.metmuseum.org/Works_of_Art/viewOnezoom.asp?dep=14&zoomFlag=0&viewmode=0&item=91%2E1%2E2064>

[2] Edited version of Bourgoin, J. Arabic Geometrical Pattern and Design. Dover Publications, 1973. 76.